

# QUARK HADRON PHASE TRANSITION AND HYBRID STARS

Sanjay K. Ghosh, S. C. Phatak and Pradip K. Sahu

Institute of Physics, Bhubaneswar-751005, INDIA.

February 9, 2008

## Abstract

We investigate the properties of hybrid stars consisting of quark matter in the core and hadron matter in outer region. The hadronic and quark matter equations of state are calculated by using nonlinear Walecka model and chiral colour dielectric (CCD) model respectively. We find that the phase transition from hadron to quark matter is possible in a narrow range of the parameters of nonlinear Walecka and CCD models. The transition is strong or weak first order depending on the parameters used. The EOS thus obtained, is used to study the properties of hybrid stars. We find that the calculated hybrid star properties are similar to those of pure neutron stars.

# 1 Introduction

The first model of neutron stars was proposed by Tolman, Oppenheimer and Volkoff [1] in late 30's. In this model, the properties of neutron stars were calculated by assuming that the neutron matter consists of non-interacting degenerate gas of neutrons. Since then, a number of improved calculations [2]-[6], which use different nuclear equation of state (EOS), have been performed to determine the composition and properties of neutron stars. The subject of neutron stars received a new impetus after the discovery of pulsars [7, 8]. In spite of two decades of work, the EOS required in the calculation of neutron star properties is uncertain for a number of reasons. The EOS at large baryon densities sensitively depends on short range nucleon - nucleon interaction and this is one source of uncertainties. Further more, at large baryon densities, weak decay of neutrons into hyperons may be energetically allowed and the neutron matter at high densities may contain hyperons. Efforts have been made to include hyperon degrees of freedom in nuclear EOS [9]-[12]. However, it must be noted that the interaction of hyperons with other baryons is not well understood and the EOS is, therefore uncertain.

The quark structure of hadrons implies that at sufficiently large nuclear densities the nuclear matter should convert itself into quark matter. The question of transition density and the order of the transition of the nuclear matter to quark matter is not yet settled. The density at which the transition occurs, is believed to be few times nuclear matter density. The lattice calculations indicate that for nonzero quark masses the phase transition may be weak first order or second order [13]. Most of the model calculations assume it to be first order. Thus, for large enough mass of neutron star, its core may consist of quark matter. In addition, if the phase transition is first order, a part of

the core may consist of a mixed phase of quark and nuclear matter. Particularly, if, at the phase transition, the discontinuity in the baryon density between quark and baryon matters is large, a substantial fraction of neutron star may be in the mixed phase. It is therefore, interesting to investigate the properties of neutron stars having a mixture of quark and nuclear matter.

In the present work, we have studied the quark-hadron phase transition at zero temperature using two phase model and applied it to the calculation of neutron star properties. Ideally, one would like to investigate this phase transition using same model in both the phases. One such method would be the lattice QCD calculations. However, the problem of inclusion of dynamical fermions in lattice calculations is not solved and the calculations of equation of state for nonzero baryon chemical potential will not be available in near future. Thus, a number of calculations employing different models in two phases have been done [4, 6, 10]. Here, we want to present a calculation where we have used nonlinear Walecka model[10, 11, 12] for nuclear matter and chiral colour dielectric model(CCDM) for quark matter [14].

The nonlinear Walecka model has been used extensively in nuclear structure calculations. It has been observed that, with nonlinear self-interaction of the  $\sigma$  field, one obtains very good agreement with the properties of nuclei over a wide range of the periodic table [16]. Thus, it seems reasonable to use such a model to describe the hadronic phase in a region where nuclear densities are not too large in comparison with the nuclear matter density. The equation of state calculation in hadronic phase includes hyperons as well as leptons. The parameters of the nonlinear Walecka model are determined by fitting the properties of nuclear matter. However, the coupling of  $\sigma$ ,  $\rho$  and  $\omega$  mesons to hyperons is not determined by this procedure. We have therefore varied these couplings within

a reasonable range and investigated the effect of these on the phase transition and the properties of stars.

The CCDDM has been used earlier in baryon spectroscopy [17]. These calculations have shown that the model is able to explain the static properties of light baryons very well. Further more, when applied to quark matter calculation, the model yields an equation of state which is quite similar to the one obtained in lattice calculations for zero baryon chemical potential. In particular, it shows that the energy density calculated in CCDDM is close to the energy density of free quarks and gluons and the pressure decreases rapidly when the temperature is close to the transition temperature [14]. We therefore, think that CCDDM is a qualitative improvement over the bag model, which is often used to calculate the equation of state in quark matter. The model has also been used in the calculation of quark star properties [18].

The results of our calculations can be briefly summarised as follows. We find that the hadron to quark matter phase transition occurs for a narrow range of the parameter sets of the models of the two phases. Particularly, the phase transition does not occur for smaller values of nuclear compressibility as well as for certain parameter sets of the CCDDM (see later). This probably indicates that certain range of parameters of nonlinear Walecka model and/or CCDDM are physically not acceptable. The neutron stars in our model, called hybrid star, may consist of quark core, a mixed phase region and then outer part made up of neutrons. The width of the mixed phase depends on the parameter set used. Usually a stiffer EOS gives higher mass or radius for a pure neutron star. But for a hybrid star, trend is opposite. The neutron stars in our model is found to be consistent with the observational limits.

The paper is organised as follows. In Section 2, we obtain the equations of state for

nuclear and quark matter. Section 3 is devoted to the discussion on hadron-quark phase transition. The results for the neutron, quark and hybrid stars structure are discussed in Section 4 and the conclusions are presented in section 5.

## 2 Equations of State

### 2.1 Hadrons

The equation of state for hadrons is calculated in the frame work of mean field theory using the nonlinear Walecka Lagrangian given below [10, 12].

$$\begin{aligned}
\mathcal{L}(x) = & \sum_i \bar{\psi}_i (i\gamma^\mu \partial_\mu - m_i + g_{\sigma i} \sigma + g_{\omega i} \omega_\mu \gamma^\mu - g_{\rho i} \rho_\mu^a \gamma^\mu T_a) \psi_i \\
& - \frac{1}{4} \omega^{\mu\nu} \omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4} \rho_{\mu\nu}^a \rho_a^{\mu\nu} \\
& + \frac{1}{2} m_\rho^2 \rho_\mu^a \rho_a^\mu - \frac{1}{3} b m_N (g_{\sigma N} \sigma)^3 - \frac{1}{4} c (g_{\sigma N} \sigma)^4 + \sum_l \bar{\psi}_l (i\gamma^\mu \partial_\mu - m_l) \psi_l
\end{aligned} \tag{1}$$

The Lagrangian in eq(1) above includes nucleons,  $\Lambda$  and  $\Sigma^-$  hyperons (denoted by subscript  $i$ ), electrons and muons (denoted by subscript  $l$ ) and  $\sigma$ ,  $\omega$  and  $\rho$  mesons (given by  $\sigma$ ,  $\omega^\mu$  and  $\rho^{a,\mu}$  respectively). The Lagrangian includes cubic and quartic self-interactions of the  $\sigma$  field. The meson fields interact with baryons through linear coupling and the coupling constants are different for nonstrange and strange baryons. The Lagrangian defined above (without strange baryons) has been used extensively in nuclear structure calculations with success. The parameters of the model are meson-baryon coupling constants, meson masses and the coefficients of the cubic and quartic self-interactions of  $\sigma$  meson (  $b$  and  $c$  respectively). The  $\omega$  and  $\rho$  masses are chosen to be their physical masses.

The equation of state is obtained by adopting mean field ansatz. Thus in presence of baryons, the mesons develop nonzero vacuum expectation values ( $\bar{\sigma}$ ,  $\bar{\omega}$  and  $\bar{\rho}^a$  respectively). Assuming that the baryon densities are uniform, one finds that the time components of  $\bar{\omega}$  and  $\bar{\rho}^3$ , in addition to  $\bar{\sigma}$ , are nonzero. One can then define effective masses ( $\bar{m}_i$ ) and chemical potentials ( $\bar{\mu}_i$ ) for the baryons as,

$$\bar{m}_i = m_i - g_{\sigma i} \bar{\sigma} \quad (2)$$

and

$$\bar{\mu}_i = \mu_i - g_{\omega i} \bar{\omega} - I_3 g_{\rho N} \bar{\rho}^3 \quad (3)$$

where  $I_3$  is the value of the z-component of the isospin of baryon  $i$ . The Fermi momenta ( $k_i$ ) and number densities ( $n_i$ ) of the baryons are given by  $k_i^2 = \sqrt{\bar{\mu}_i^2 - \bar{m}_i^2}$  and  $n_i = \frac{k_i^3}{3\pi^2}$ . For leptons, the Fermi momenta and number densities are given by  $k_l = \sqrt{\mu_l^2 - m_e^2}$  and  $n_l = \frac{k_l^3}{3\pi^2}$ .

The condition of equilibrium under weak interactions (assuming that neutrinos are not degenerate) give the following relations between baryon and lepton chemical potentials

$$\begin{aligned} \mu_p &= \mu_n - \mu_e, & \mu_\Lambda &= \mu_n, \\ \mu_{\Sigma^-} &= \mu_n + \mu_p, & \mu_\mu &= \mu_e \end{aligned} \quad (4)$$

and charge neutrality gives,

$$n_p = n_e + n_\mu + n_\Sigma \quad (5)$$

In addition,  $n_B = n_n + n_p + n_\Lambda + n_\Sigma$  is the total baryon density and  $\mu_B = \mu_n$  is defined as the baryon chemical potential.

The mean field values of  $\bar{\sigma}$ ,  $\bar{\omega}_0$  and  $\bar{\rho}_0^3$  are determined by minimizing the energy at fixed baryon density. Then the EOS is calculated using the expression for pressure  $P$  and energy density  $E$  as given below.

$$P = \frac{1}{2}m_\omega^2\bar{\omega}_0^2 + \frac{1}{2}m_\rho^2(\bar{\rho}_0^3)^2 - \frac{1}{2}m_\sigma^2\bar{\sigma}^2 - \frac{1}{3}bm_N(g_{\sigma N}\bar{\sigma})^3 - \frac{1}{4}c(g_{\sigma N}\bar{\sigma})^4 + \sum_i P_{FG}(\bar{m}_i, \bar{\mu}_i) + \sum_l P_{FG}(m_l, \mu_l) \quad (6)$$

$$E = \frac{1}{2}m_\omega^2\bar{\omega}_0^2 + \frac{1}{2}m_\rho^2(\bar{\rho}_0^3)^2 + \frac{1}{2}m_\sigma^2\bar{\sigma}^2 + \frac{1}{3}bm_N(g_{\sigma N}\bar{\sigma})^3 + \frac{1}{4}c(g_{\sigma N}\bar{\sigma})^4 + \sum_i E_{FG}(\bar{m}_i, \bar{\mu}_i) + \sum_l E_{FG}(m_l, \mu_l) \quad (7)$$

In the above equations  $P_{FG}$  and  $E_{FG}$  are the relativistic non-interacting pressure and energy density of the fermions. Nonlinear Walecka model has eight parameters out of which five are determined by the properties of nuclear matter. These are nucleon coupling to scalar ( $\frac{g_\sigma}{m_\sigma}$ ), vector ( $\frac{g_\rho}{m_\rho}$ ) and isovector mesons ( $\frac{g_\omega}{m_\omega}$ ) and the two coefficients in scalar self interaction i.e.  $b$  and  $c$ . These are obtained by fitting saturation values of the binding energy/nucleon ( $-16MeV$ ), baryon density ( $0.15fm^{-3}$ ), symmetry energy coefficient ( $32.5MeV$ ), Landau mass ( $0.83m_N$ ). The nuclear compressibility is somewhat uncertain and therefore, we have varied it between  $250MeV - 350MeV$ . The values of these parameters along with compressibility are presented in Table 1.

The other three parameters are coupling constants of hyperon-meson interaction and are not well known. These cannot be determined from nuclear matter properties since the nuclear matter does not contain hyperons. Further more, properties of hypernuclei do not fix these parameters in a unique way. In the literature, a number of choices have been made. These along with our choice are listed below.

1. choose them to be same as nucleon-meson coupling constants (Universal coupling ) [10],
2. choose them to be  $\sqrt{2/3}$  times nucleon-meson coupling constants [11],
3. choose them to be  $1/3$  times nucleon-meson coupling constants [12]
4. It is claimed that the second choice follows from quark counting rule. However, using SU(6) quark wavefunctions of baryons and assuming that the mesons do not couple to strange quarks, we find that the ratio of meson-hyperon and meson-nucleon couplings are  $2/3$ ,  $2/3$  and  $1$  for  $\sigma$ ,  $\omega$  and  $\rho$  mesons respectively.

In our calculation, we have used the above couplings to study the nuclear EOS and star characteristics.

## 2.2 Quarks

The Colour Dielectric Model (CDM) is based on the idea of Nelson and Patkos [19]. In this model, one generates the confinement of quarks and gluons dynamically through the interaction of these fields with scalar field. In the present work, we have used the chiral extension [14] (CCDM) of this model to calculate quark matter EOS. The Lagrangian density of CCDM is given by

$$\begin{aligned}
\mathcal{L}(x) = & \bar{\psi}(x)\{i\gamma^\mu\partial_\mu - (m_0 + m/\chi(x)U_5) + (1/2)g\gamma_\mu\lambda_a A_\mu^a(x)\}\psi \\
& + f_\pi^2/4Tr(\partial_\mu U\partial^\mu U^\dagger) - 1/2m_\phi^2\phi^2(x) \\
& - (1/4)\chi^4(x)(F_{\mu\nu}^a(x))^2 + (1/2)\sigma_v^2(\partial_\mu\chi(x))^2 - U(\chi)
\end{aligned} \tag{8}$$

where  $U = e^{i\lambda_a\phi^a/f_\pi}$  and  $U_5 = e^{i\lambda_a\phi^a\gamma_5/f_\pi}$ ,  $\psi(x)$ ,  $A_\mu(x)$ ,  $\chi(x)$  and  $\phi(x)$  are quark, gluon, scalar ( colour dielectric ) and meson fields respectively,  $m_\phi$  and  $m$  are the meson and



quark masses,  $f_\pi$  is the pion decay constant,  $F_{\mu\nu}(x)$  is the usual colour electromagnetic field tensor,  $g$  is the colour coupling constant and  $\lambda_a$  are the Gell-Mann matrices. The flavour symmetry breaking is incorporated in the Lagrangian through the quark mass term  $(m_0 + m/\chi U_5)$ , with  $m_0 = 0$  for  $u$  and  $d$  quarks. So the masses of  $u$ ,  $d$  and  $s$  quarks are  $m$ ,  $m$  and  $m_0 + m$  respectively. The self interaction  $U(\chi)$  of the scalar field is assumed to be of the form

$$\alpha B \chi^2(x) [1 - 2(1 - 2/\alpha)\chi(x) + (1 - 3/\alpha)\chi^2(x)] \quad (9)$$

so that  $U(\chi)$  has an absolute minimum at  $\chi = 0$  and a secondary minimum at  $\chi = 1$ . The interaction of the scalar field with quark and gluon fields is such that quarks and gluons can not exist in the region where  $\chi = 0$ . In the limit of vanishing meson mass, the Lagrangian of eqn.(8) is invariant under chiral transformations of quark and meson fields.

The calculation of equation of state proceeds as follows. We assume that, in presence of nonzero quark/anti-quark densities, the square of meson fields may develop nonzero vacuum expectation values  $\langle \phi^2 \rangle$  [14, 18]. This assumption is analogous to the assumption that, in linear  $\sigma$  model [15], the  $\sigma$  field acquires nonzero vacuum expectation. In [14], it has been shown that this occurs when the quark density exceeds certain critical value and one of the effects of nonzero  $\langle \phi^2 \rangle$  is that the effective quark masses decrease with the increase in  $\langle \phi^2 \rangle$ . Thus at large quark densities, one obtains an equation of state similar to the equation of state of free quarks and gluons. Further more, we adopt mean field approximation to calculate the colour dielectric field ( $\chi$ ) in quark matter. With these assumptions, the thermodynamic potential for the quark matter is given by,

$$\Omega = \frac{1}{4\pi^2} \sum_i \left\{ [\mu_i k_i (\mu_i^2 - \frac{5}{2} m_i^{*2}) + \frac{3}{2} \ln(\frac{\mu_i + k_i}{m_i^*})] \right\}$$

$$- \frac{\alpha_s}{\pi} \left[ \frac{3}{2} (\mu_i k_i - m_i^{*2} \ln(\frac{\mu_i + k_i}{m_i^*}))^2 - k_i^4 \right] \} \quad (10)$$

where  $i = u, d, s$ . Also,  $k_i = \sqrt{\mu_i^2 - m_i^{*2}}$  and this becomes equal to Fermi momentum for  $\alpha_s = 0$ . Here  $\Omega$  is calculated upto second order in quark-gluon interaction.

In addition, chemical equilibrium under weak decay and charge neutrality imply,

$$\begin{aligned} \mu_d &= \mu_u + \mu_e \\ \mu_s &= \mu_u + \mu_e \end{aligned} \quad (11)$$

and

$$2/3n_u - 1/3n_d - 1/3n_s - n_e = 0 \quad (12)$$

The baryon density  $n_B = 1/3 \sum_i (n_i)$  where  $i = u, d, s$  and baryon chemical potential is defined as  $\mu_B = \mu_d + \mu_u + \mu_s$ .

The mean field values of  $\chi$  and  $F_\phi$  are calculated by minimising  $\Omega$ . Equivalently, one can solve the equations of motion for  $\chi$ ,  $F_\pi$ ,  $F_K$  and  $F_\eta$  as obtained from the Lagrangian. We find that  $\langle \vec{\pi}^2 \rangle$  alone develops non zero value in the quark matter.  $\langle K^2 \rangle$  and  $\langle \eta^2 \rangle$  remain zero throughout the range of densities considered. This implies that strange quark mass remain constant in the medium. On the other hand,  $u$  and  $d$  quark masses change in the medium. The pressure  $P$  and energy density  $E$  of the quark matter are calculated using the relations,  $P = -\Omega$  and  $E = \Omega + \mu_i n_i$ . Here, the number density of the quark of  $i$ th type is given by  $n_i = -(\partial\Omega/\partial\mu_i)$

The parameters of the CCDDM are  $u$  and  $d$  quark mass, strange quark mass, strong coupling constant  $\alpha_s$ , bag pressure  $B$  and  $\alpha$ . These are obtained by fitting the baryon masses. Earlier calculations [17] show that these are not determined uniquely by the fitting procedure. In particular, it has been found that good fits to baryon masses are

obtained for  $0.6\text{GeV} \leq m_{GB} \leq 3\text{GeV}$ ,  $m_q(u, d) \leq 125\text{MeV}$ ,  $m_q(s) \sim 300\text{MeV}$  and  $B^{1/4} \leq 150\text{MeV}$ , where glueball mass  $m_{GB}$  is defined as  $m_{GB}^2 = 2B\alpha/\sigma_v^2$ . The fits are better for lower values of  $m_{GB}$ ,  $m_q(u, d)$  and  $B$ . Since we do not consider scalar field excitations, the quark matter EOS does not depend on  $m_{GB}$ .

### 3 Hadron - Quark Phase Transition

Having determined the equations of state of neutron and quark matter, the phase transition point is determined by adopting Gibb's criterion. That is, the point at which the free energies ( or pressure ), for given chemical potential, are equal gives the phase transition point. In the present context, the baryon and electron chemical potentials (  $\mu_B$  and  $\mu_e$  respectively ) are the two independent chemical potentials. However, if we assume the charge neutrality,  $\mu_e$  is no more independent. Then the crossing of the pressure curves for two phases in  $P$ - $\mu_B$  plane gives the phase transition point. Glendenning [11], on the other hand considered a case where, in the mixed phase, neutron and quark matters are not charge neutral but the mixture as a whole is. This aspect has been studied further by Heiselberg et.al. in ref [20]. We are not considering this situation here.

For a first order transition, the derivatives of  $P$ - $\mu_B$  curve for the two phases at the phase transition point are not equal and the difference in the two derivatives gives the discontinuity in the density of the two phases at the transition point. The two phases coexist in this range of density. The latent heat of transition is given by the difference in the energy densities of the two phases at the critical point. As mentioned earlier, the lattice QCD calculations indicate that hadron - quark phase transition may be weakly first order or second order. In a calculation, such as ours, we will necessarily have a

first order transition since two different models are employed to calculate the properties of quark and hadron phases. However, our calculation shows that the phase transition can be made weaker by varying the compressibility of the nuclear matter as well as the meson- strange baryon coupling within reasonable limits. For example, Table 2 shows that the latent heat can be reduced from  $154\text{MeV}/fm^3$  to  $96\text{MeV}/fm^3$  by changing the compressibility from  $250\text{MeV}$  to  $300\text{MeV}$ . Thus, it is possible to mimick the second order phase transition in this manner.

Our calculation shows that as the nuclear compressibility is increased from  $250\text{MeV}$ , the chemical potential ( and baryon density ) at which the phase transition occurs decreases. ( see Fig. 1) This can be understood as follows. For a given parametrization, the nucleon- nucleon repulsive interaction increases with compressibility and hence the slope of the  $P - \mu_B$  curve at the transition point i.e. the baryon density decreases. The dependence of latent heat on compressibility is given in Table 2.

As mentioned earlier, the strange quark - meson coupling constant in hadron phase is not well known. We have therefore, varied this coupling constant as mentioned in earlier section and investigated its effect on the properties of phase transition. We find that as the coupling constant is reduced from the universal coupling (  $g_{H\alpha}/g_{N\alpha}=1$ ,  $\alpha = \sigma, \omega$  or  $\rho$  and  $H = \Lambda$  or  $\Sigma^-$  ) the slope of the  $P - \mu_B$  curve ( Fig.2 - 5) increase which leads to the increase in transition density and as well as reduction in the latent heat (Fig.6 and Table 2). The EOS becomes softer with decrease in hyperon couplings. This can be understood as follows. With decrease in hyperon couplings, it is energetically favourable to convert nucleons into hyperons as hyperons do not feel the predominantly repulsive force. As a result with decreasing coupling more and more hyperons get populated thereby reducing the energy further.

As mentioned in the previous section, the quark matter EOS has been obtained by using the parameter sets which fit the baryon masses. We find that the transition density for hadron quark phase transition decreases rapidly with the decrease in  $B$ . For  $B^{1/4} \leq 140 \text{ MeV}$ , the transition density is smaller than the nuclear matter density. This is unphysical, since it implies that we should have quark matter at nuclear matter densities. This happens because the contribution of the dielectric field to the pressure, which is negative, is proportional to  $B$ . So for smaller  $B$ , the pressure of the quark matter increase and this makes the quark matter stable at lower chemical potentials and densities. This type of behaviour has also been noticed for the bag model EOS where one requires  $B^{1/4} \sim 150 \text{ MeV}$  [6]. At this point, we would like to note that  $B$  can not be increased arbitrarily if one insists on a reasonable fit to baryon masses. Thus in CCDDM, a very restricted parameter sets give reasonable values for transition densities. One such set ( $B^{1/4} = 152.1 \text{ MeV}$ ,  $m_{q(u,d)} = 91.6 \text{ MeV}$ ,  $m_{q(s)} = 294.9 \text{ MeV}$   $\alpha = 36$  and strong coupling  $\alpha_s = 0.08$ ) has been used in the calculations reported here.

Finally, let us consider the change in strangeness fraction at the transition point. The strangeness fraction in quark and nuclear matter are defined by the ratio strange quark density/ baryon density and strange hadron density/baryon density respectively. Note that for equal mass quarks and  $\Lambda$  matter this ratio is unity. We find that generally at the transition point the strangeness fraction is larger in quark matter. This is shown in Fig.7, where strange fraction is plotted as a function of baryon density. Note that in the coexistence region, the system consist of a mixture of quark and nuclear matter. The strangeness fractions in the mixed phase are calculated using the linear relation  $f_s(M) = (1 - \chi)f_s(H) + \chi f_s(Q)$ , where  $\chi$  is the concentration of the quark matter in the mixed phase and  $f_s$  is the strangeness fraction,  $M$ ,  $H$ , and  $Q$  denoting the

mixed, hadronic and quark phase respectively. Similar relation is used to calculate the corresponding baryon density as well. It is evident from the graph that for a large mixed phase region, the jump in the strangeness fraction from hadronic to quark phase is larger.

The presence of considerable amount of strangeness in the hadronic sector in the form of hyperons has important consequences on the mechanism of phase transition and core of the neutron stars. Some of the possible mechanisms are discussed in ref [Alcock, Farhi and Olinto [21]]. The conversion via two flavour has been considered in ref [22], where as in ref [23], Olinto has started with the assumption that strange matter has been seeded into neutron star from outside. On the other hand, presence of considerable amount of hyperons near the transition point suggest that inside a cold neutron star conversion from neutron to strange quark matter may occur through the clustering of  $\Lambda$ 's [21]. For a hot neutron star the conversion to strange matter will occur due to thermal  $\Lambda$ 's. Also, along with these processes, there will be usual strangeness changing weak decay to convert excess  $d$  quarks to  $s$  quarks. We believe that these points need further investigation.

## 4 Neutron star models

Quark cores can exist inside a neutron/hybrid star only for a narrow range of parameter sets in our model. The extent of quark core will be higher for larger compressibility and larger hyperon couplings.

The structure of a neutron star is characterised by its mass and radius. Additional parameters of interest are the moment of inertia, the surface red shift  $z$  and the rela-

tivistic Keplerian rotation period  $P_K$  defined as [4]:

$$P_K = 0.026 \sqrt{\frac{(R/km)^3}{(M/M_\odot)}} [ms] \quad (13)$$

as a function of the central density  $\rho_c$  of the star. These are important for the dynamics and transport properties of pulsars.

The equations that describe the hydrostatic equilibrium of degenerate stars without rotation in general relativity is called Tolman-Oppenheimer-Volkoff (TOV) equations, which is given in ref [3, 18, 24, 25]. These equations can be numerically integrated, for a given central density, to obtain the radius  $R$  and the gravitational mass  $M$  of the star. The moment of inertia  $I$  of the rotating neutron star, is also calculated [see ref [3, 25]] for the corresponding central density. To integrate the TOV equations, one needs to know the equation of state  $P(\rho)$ , for the entire expected density range of neutron star, starting from the higher density at the center to the surface density. The composite equation of state for the entire neutron star density span, was constructed by joining the nonlinear Walecka hadronic equation of state (eqn.6-7) to that given by (a) Negle and Vautherin [26] for the density range  $10^{14}$  to  $5 \times 10^{10} \text{ gm/cm}^3$ , (b) Baym, Pethick and Sutherland [27] for the range  $5 \times 10^{10}$  to  $10^3 \text{ gm/cm}^3$  and (c) Feynman, Metropolis and Teller [28] for densities less than  $10^3 \text{ gm/cm}^3$ .

The results for star structure parameters are listed in Table 3. Fig.8, and Fig.9 show plots of mass vs central density and mass vs moment of inertia respectively. We have plotted the curves for hadronic EOS with hyperon couplings (1), (2) and (4) as given in section 2.1 and fixed compressibility  $300 \text{ MeV}$ . In case of quark matter, we have used the interacting CCDM with the parameter set discussed in section 3. For neutron stars a stiffer EOS results in a larger maximum mass and radius [4, 12]. So for heavier stars, one

needs larger compression constant  $K$  or larger hyperon couplings. Similarly for quark stars, one needs larger  $B$  or  $\alpha_s$ . This dependence is reversed for hybrid stars where a more repulsive interaction yields lighter stars as can be seen in Fig.8 and Table 3. This is because a stiffer EOS implies lower critical baryon densities in the hadronic sector and hence larger quark cores. Since quark EOS yield smaller maximum masses, this reduces the maximum mass of the hybrid stars (Fig.8 and Table 3).

Here we would like to mention that the most accurately determined mass is that of PSR1913+ 16 with  $M/M_\odot = 1.44 \pm 0.003$  [8]. The observational lower limit on the moment of inertia [29] is  $I = 40M_\odot km^2 \simeq 8 \times 10^{44} gm/cm^3$ . The red shift, which is not measured for any neutron star with known mass, seem to lie in the range 0.2 - 0.5. In principle, the above observational results on neutron stars should put some constraint on the EOS. But in practice, most of the EOS reproduce consistent results as found by other authors as well [4, 12]. This can also be seen from our results for characteristics of the maximum mass as given in Table 3. Table 3 contains the results for neutron stars models *HM I*, *HM II* and *HM III* corresponding to three hyperon couplings ((1), (2) and (4), see section 2.1) with compressibility  $300MeV$  and for hybrid stars models *Hybrid I*, *Hybrid II* and *Hybrid III* respectively. In our model, the maximum gravitational mass for stable non-rotating neutron stars are in the range  $1.86M_\odot - 1.6M_\odot$ . This decrease in maximum mass, is due to softness of EOS. That is because of decrease in hyperon couplings as explained earlier. The corresponding radius increases from  $10.69km - 11.00km$ , whereas, red shift and moment of inertia decrease from  $0.44 - 0.32$  and  $1.76 \times 10^{45} g cm^2 - 1.52 \times 10^{45} g cm^2$  respectively. Similarly, the hybrid star maximum mass corresponding to the three hyperon couplings are in the range  $1.47M_\odot - 1.49M_\odot$ . The corresponding variation of radius, red shift and moment of



inertia are  $12.36km - 12.0km$ ,  $0.242 - 0.256$  and  $1.75 \times 10^{45} g cm^2 - 1.71 \times 10^{45} g cm^2$ . The small variation of hybrid star properties implies that hadronic EOS does not have strong influence on the masses and radii of the hybrid star as observed by other authors as well [4]. We have also calculated the Keplerian rotation period. The minimal rotation periods for neutron stars are in the range  $0.66ms - 0.75ms$ . The corresponding range for hybrid stars is  $0.93ms - 0.89ms$ . So the periods for both neutron and hybrid stars, in our model, are comparable with the limits obtained by other authors [4]. Thus, the general characteristics of stable star, in our model, are compatible with the observational estimates.

The post glitch data set from the vela pulsar indicates that crust superfluid comprises of about  $3.4 \times 10^{-2}$  of the stars moment of inertia [30]. This is lower bound on the fractional moment of inertia of the entire crust. We have calculated  $\alpha = I_p/I$ , where  $I_p$  is the moment of inertia of the pinned superfluid region and  $I$  is the total moment of inertia of the star. This region is defined as the radial extent corresponding to the density  $2 \times 10^{14} - 2 \times 10^{13} gm/cm^3$ . Table 3 shows that the value of  $\alpha$  is sensitive to the values of hyperon couplings. The purpose of this calculation is to compare an observational feature of pulsar glitches with predicted theoretical values of neutron and hybrid stars in the present model.

## 5 Summary

We have described the hadron - quark phase transition at zero temperature in the frame work of relativistic mean field theory for hadronic sector and chiral Colour Dielectric model for quark sector. We have studied the hadronic EOS and phase transition by

varying the compressibility from  $250\text{MeV} - 350\text{MeV}$ . Using a simple quark counting rule, we get a hyperon coupling which is different compared to those used by other authors [10, 11, 12]. We have given results for two other couplings found in the literature as well. We find that critical  $\mu_B$  and critical  $n_B$  can be reduced by increasing the hyperon coupling or compressibility. Moreover, latent heat can also be reduced by decreasing the hyperon coupling or varying the compressibility. This further implies a weaker first order transition. In quark sector, we have taken the parameter sets from baryon spectroscopy. We find that only for a very narrow range of parameters in CCDDM, one can get phase transition. In CCDDM, the quark matter EOS depends sensitively on quark masses, strong coupling and bag pressure. For  $B^{1/4} < 140\text{MeV}$  the transition density is less than the nuclear matter density which is unphysical. Here we would like to mention that in our calculation, it is not possible to change any of the parameters arbitrarily, as it would destroy the fit of baryon masses.

We find that a considerable amount of strangeness is present in the hadronic phase at the transition point. Also there is a large change in the strangeness fraction from hadronic to quark phase. Several authors [22] have studied the production rates of strangeness during the phase transition from nuclear to quark matter. But as predicted by Alcock et.al. [21], the rate of transition to quark matter may be much faster, if strangeness is already present in the hadronic phase. It will be interesting to study the mechanism of transition to quark phase and the rate of strangeness production in such circumstances.

The neutron star characteristics are calculated for different EOS obtained from Walecka models and CCDDM. It is clear that the mass limits for stars are sensitive to the parameter sets. We find that CCDDM gives softer EOS compared to bag models and

hence lower masses and radii. Hybrid star masses are lower compared to pure neutron stars because of the quark cores. We found that the stars properties are not much sensitive to the composition of stars. Also these characteristics obey the observationally inferred limits. So, it is difficult to distinguish between the stars of different composition observationally. Hence the most massive pulsars observed so far could possibly either a neutron star or a hybrid star. We do not get the layered structure as predicted in reference [31]. In fact, we find that parameter sets which predict a stable quark phase at lower densities, also predict a unstable isospin symmetric nuclear matter at lower densities. Here we would like to point out that the nuclear to quark matter phase transition can happen through some intermediate phases as well. The nuclear matter may go over to a pion or kaon [32] condensate phase through a second order transition which then goes over to quark phase at higher densities. In such case the transition to quark phase happens at much higher densities. Some other authors have tried to explore the possibility of second order transition [12], which we do not consider here.

## References

- [1] J. R. Oppenheimer and R. Serber, Phys. Rev. **54** (1938) 540; J. R. Oppenheimer and G. M. Volkoff, **55** (1939) 374; R. C. Tolman, Phys. Rev. **55** (1939) 364.
- [2] R. B. Wiringa, V. Fiks and A. Fabrocini, Phys. Rev. **C38** (1988) 1010; R. B. Wiringa, Rev. of Mod. Phys. **65** (1993) 231, J. N. Bachall and R. A. Wolf, Phys. Rev. **B140** (1965) 1445.
- [3] P. K. Sahu, R. Basu and B. Datta, Astrophys. J. **416** (1993) 267.
- [4] A. Rosenhaur, E. F. Staubo and L. P. Csernai, Nucl. Phys. **A540** (1992) 630; Z. Phys. **A342** (1992) 235.
- [5] M. Kutschera and A. Koltroz, Astrophys. J. **419** (1993) 752.
- [6] B. D. Serot and H. Uechi, Ann. of Phys. **179** (1987) 272.
- [7] A. Hewish, S. J. Bell, J. D. H. Pilkington, P.F. Scott and R. A. Collins, Nature **217** (1968) 709; P. C. Joss and S. A. Rappaport, Ann. Rev. of Astr. and Astrophys. **22** (1984) 537; D. C. Baker, S. Kulkarni, C. Heiles, M. M. Davis and W. M. Goss, Nature **300** (1982) 615.
- [8] J. H. Taylor and J. M. Weisberg, Astrophys. J. **345** (1989) 434.
- [9] V. R. Pandharipande, Nucl. Phys. **A178** (1971) 213; A. Bethe and M. Johnson, Nucl. Phys. **A230** (1974) 1; R. L. Bowers, A. M. Gleeson and R. D. Pedigo, Phys. Rev. **D12** (1975) 3056.
- [10] N. K. Glendenning, Astrophys. J. **293** (1985) 470.

- [11] N. K. Glendenning, F. Weber and S. A. Moszkowski, Phys. Rev. **C45** (1992) 844.
- [12] J. I. Kapusta and K. A. Olive, Phys. Rev. Lett. **64** (1990) 13; J. Ellis, J. I. Kapusta and K. A. Olive, Nucl. Phys. **B348** (1991) 345.
- [13] J. Engels, F. Karsch, H. Satz and I. Montvay, Nucl. Phys. **B205** (1982) 545; J. P. Blaizot, Nucl. Phys. **A566** (1994) 333c.
- [14] S. K. Ghosh and S. C. Phatak, J. Phys. **G18** (1992) 755.
- [15] W. Broniowski, M. Cibej, M. Kutschera and M. Rossina, Phys. Rev. **D41** (1990) 285.
- [16] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. **16** (1986) 1; S. K. Patra and C. R. Praharaj, Europhys. Lett. **20** (1992) 87.
- [17] S. Sahu and S. C. Phatak, Mod. Phys. Lett. **A7** (1992) 709.
- [18] S. K. Ghosh and P. K. Sahu, Int. Jour. Mod. Phys. **E2** (1993) 575.
- [19] H. B. Nielson and A. Patkos, Nucl. Phys. **B197** (1982) 139.
- [20] E. F. Staubo, H. Heiselberg and C. J. Pethick, Nucl. Phys. **A566** (1994) 577c; H. Heiselberg, C. J. Pethick and E. F. Staubo, Phys. Rev Lett. **70** (1993) 1355.
- [21] C. Alcock, E. Farhi and A. Olinto, Astrophys. J. **310** (1986) 261.
- [22] Z. Dai, T. Lu and Q. Peng, Phys. Lett. **B319** (1993) 199; H. Heiselberg, G. Baym and C. J. Pethick, Nucl. Phys. **24B** (1991) 144; M. C. Olsen and J. Madsen, Nucl. Phys. **24B** (1991) 170.

- [23] A. Olinto, Phys. Lett. **B192** (1987) 71.
- [24] B. Datta, P. K. Sahu, J. D. Anand and A. Goyal, Phys. Lett. **B283** (1992) 313.
- [25] P. K. Sahu, Ph. D. Thesis, Institute of Physics, Bhubaneswar (1994).
- [26] J. W. Negle, D. Vautherin, Nucl. Phys. **A207** (1973) 298.
- [27] G. Baym, C. J. Pethick and P. G. Sutherland, Astrophys. J. **170** (1971) 299.
- [28] R. P. Feynmann, N. Metropolis and E. Teller, Phys. Rev. **75** (1949) 1561.
- [29] J. Diaz Allonso and J. M. I. Cabanell, Astrophys. J. **291** (1985) 308.
- [30] B. Datta and M. A. Alpar, Astrn. Astrophys. **275** (1993) 210.
- [31] P. A. Carinhas, Preprint WISC- MIL- 92- TH- 15.
- [32] V. Thorsson, M. Prakash and J. M. Lattimer, Nucl. Phys. **A572** (1994) 693.

Table 1: Coupling constants for several compressibility  $K$  and  $B/A=-16$   $MeV$ ,  $\rho = 0.15$   $fm^{-3}$ ,  $a_{sym} = 32$   $MeV$  and  $m^*_{sat}/m = 0.8$

$K$ ( $MeV$ )	$(g_s/m_s)^2$ ( $fm^2$ )	$(g_w/m_w)^2$ ( $fm^2$ )	$(g_\rho/m_\rho)^2$ ( $fm^2$ )	$b$	$c$
250	9.216	4.356	5.025	0.008209	0.007385
300	8.492	4.356	5.025	0.002084	0.02780
350	7.820	4.356	5.025	-0.004618	0.05015

Table 2: Characteristics at the critical point. The columns correspond to compressibility ( $K$ ), hyperon couplings ( $HC$ ), critical pressure ( $P_c$ ), critical chemical potential ( $\mu_c$ ), energy of the hadronic phase at critical point ( $E_{cH}$ ), latent heat ( $L$ ), baryon density in the hadronic phase at critical point ( $n_{B(cH)}$ ) and baryon density width of the mixed phase ( $\Delta n_B$ )

$K$ ( $MeV$ )	$HC$	$P_c$ $MeV/fm^3$	$\mu_c$ $MeV$	$E_{cH}$ ( $MeV/fm^3$ )	$L$ $MeV/fm^3$	$n_{B(cH)}$ $fm^{-3}$	$\Delta n_B$ $fm^{-3}$
300.	(1)	62.2	1179.5	490.7	123.2	0.47	0.11
	(2)	78.5	1200.0	581.2	96.4	0.53	0.09
	(4)	75.0	1194.0	603.7	58.6	0.55	0.06
250.	(2)	269.3	1428.0	1236.6	153.6	1.0	0.11
350.	(2)	58.0	1163.5	480.8	128.5	0.45	0.11

Table 3: Star characteristics for different models. Here  $IQM$  for interacting quark matter and  $HM$  for hadronic matter.

$\rho_c$ ( $g\ cm^{-3}$ )	$R$ ( $km$ )	$M/M_\odot$	$z$	$I$ ( $g\ cm^2$ )	$P_K$ $ms$	$I_p/I$	<i>Model</i>
$3.60 \times 10^{15}$	8.14	1.38	0.41	$8.22 \times 10^{44}$	0.52	—	<i>IQM</i>
$2.50 \times 10^{15}$	10.69	1.86	0.44	$1.76 \times 10^{45}$	0.66	$2.98 \times 10^{-2}$	<i>HM I</i>
$2.50 \times 10^{15}$	10.93	1.72	0.37	$1.52 \times 10^{45}$	0.72	$6.10 \times 10^{-2}$	<i>HM II</i>
$2.00 \times 10^{15}$	11.00	1.60	0.32	$1.52 \times 10^{45}$	0.75	$2.68 \times 10^{-2}$	<i>HM III</i>
$0.84 \times 10^{15}$	12.36	1.47	0.24	$1.75 \times 10^{45}$	0.93	$2.94 \times 10^{-2}$	<i>Hybrid I</i> ( <i>IQM+HM I</i> )
$0.99 \times 10^{15}$	12.94	1.49	0.23	$1.72 \times 10^{45}$	0.99	$5.4 \times 10^{-2}$	<i>Hybrid II</i> ( <i>IQM+HM II</i> )
$1.03 \times 10^{15}$	12.00	1.49	0.26	$1.71 \times 10^{45}$	0.89	$2.39 \times 10^{-2}$	<i>Hybrid III</i> ( <i>IQM+HM III</i> )



## FIGURE CAPTIONS

Figure 1: Pressure vs. chemical potential **(a)** Interacting quark matter ( $Q.M.$ ), **(b)** Hadronic matter ( $H.M.$ ),  $K= 250MeV$ , hyperon coupling ( $HC$ ) (2), **(c)**  $H.M.$ ,  $K= 300MeV$ ,  $HC$  (2), **(d)**  $H.M.$ ,  $K= 350MeV$ ,  $HC$  (2) and **(e)** Bag model with bag constant  $B^{1/4}= 152.1MeV$

Figure 2: Pressure vs. chemical potential **(a)**  $Q.M.$  **(b)**  $H.M.$ ,  $K= 300MeV$ ,  $HC$  (1)

Figure 3: Pressure vs. chemical potential **(a)**  $Q.M.$  **(b)**  $H.M.$ ,  $K= 300MeV$ ,  $HC$  (2)

Figure 4: Pressure vs. chemical potential **(a)**  $Q.M.$  **(b)**  $H.M.$ ,  $K= 300MeV$ ,  $HC$  (4)

Figure 5: Pressure vs. chemical potential **(a)**  $Q.M.$  **(b)**  $H.M.$ ,  $K= 300MeV$ ,  $HC= 0.75$

Figure 6: EOS with first order phase transition from  $H.M.$  to  $Q.M.$  with compressibility  $300MeV$  **(a)**  $HC$  (1), **(b)**  $HC$  (2) and **(c)**  $HC$  (4)

Figure 7: Variation of strangeness fraction with baryon density for the EOS in Fig.6, **(a)**  $HC$  (1), **(b)**  $HC$  (2) and **(c)**  $HC$  (4)

Figure 8: Mass vs. central density of neutron ( $K= 300MeV$ ), quark and hybrid stars **(a)** neutron star (N.S.),  $HC$  (1), **(b)** N.S.,  $HC$  (2), **(c)** N.S.,  $HC$  (4), **(d)** quark star . Corresponding curves for hybrid star are denoted by *Hybrid I*, *Hybrid II* and *Hybrid III* respectively.

Figure 9: Moment of inertia vs. mass of neutron ( $K= 300MeV$ ), quark and hybrid stars **(a)** N.S.,  $HC$  (1), **(b)** N.S.,  $HC$  (2), **(c)** N.S.,  $HC$  (4), **(d)** quark star . Corresponding curves for hybrid star are denoted by *Hybrid I*, *Hybrid II* and *Hybrid III* respectively.

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/nucl-th/9407009v1>

This figure "fig2-1.png" is available in "png" format from:

<http://arXiv.org/ps/nucl-th/9407009v1>

This figure "fig1-2.png" is available in "png" format from:

<http://arXiv.org/ps/nucl-th/9407009v1>

This figure "fig2-2.png" is available in "png" format from:

<http://arXiv.org/ps/nucl-th/9407009v1>

This figure "fig2-3.png" is available in "png" format from:

<http://arXiv.org/ps/nucl-th/9407009v1>